Closing Wed: HW_1A,1B,1C
Closing next Wed: HW_2A,2B,2C

Entry Task:
Draw a picture and find the following signed areas:

1. $\int_{0}^{4} 3\mathrm{x}\, dx$

2. $\int_{-2}^{4} 3\mathrm{w}\, dw$

3. $\int_{-3}^{0} \sqrt{9-\mathrm{t}^2}\, dt$

Note on quick bounds (HW_1C: 9,10)

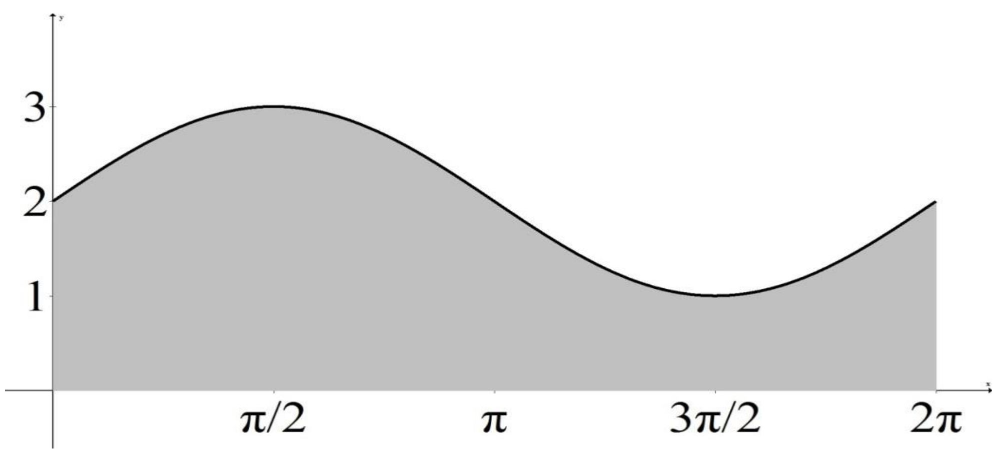
$$m(b-a) \leq \int_a^b \mathrm{f(x)}dx \leq M(b-a)$$

Example: Consider

$$\int_0^{2\pi} \sin(x) + 2\,dx$$

Let M = "max of integrand" =

Let m = "min of integrand" =



5.3 The Fundamental Theorem of Calculus (FTOC)

Motivational Task:

Consider the function *f(t) = 3t*. Draw the graph and using area formulas you know, compute:

1. $\int_{0}^{1} f(t)dt$

2. $\int_{0}^{10} f(t)dt$

3. $g(x) = \int_{0}^{x} f(t)dt$

Any observations?

Fundamental Theorem of Calculus (Part 1): *Areas under graphs are antiderivatives!*

$$\frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$$

That is, for any constant a, the "accumulated signed area" formula

$$F(x) = \int_a^x f(t)dt$$

is an antiderivative of $f(x)$.

Motivational Task:

Again, consider the function *f(t) = 3t*.
Using the area of the triangle again, simplify, then differentiate:

$$1.\ h(x) = \int_{0}^{1+x^3} f(t)dt \ , h'(x) = ?$$

$$2.\ k(x) = \int_{x^2}^{1+x^3} f(t)dt \ , k'(x) = ?$$

Any observations?

General form of FTOC (Part 1):

$$\frac{d}{dx}\left(\int_{g(x)}^{h(x)} f(t)dt\right) = f\big(h(x)\big)h'(x) - f\big(g(x)\big)g'(x)$$

Fundamental Theorem of Calculus (Part 2):

If F(x) any antiderivative of f(x),

$$\int_{a}^{b} f(x)dx = F(b) - F(a)$$